Lesson 4: Patterns—Linear, Quadratic, and Exponential

Time: Two to three class periods (50 minutes)

Grade-Level Expectations Addressed:

(A1B10) Generalize patterns using explicitly or recursively defined functions.
(A1C10) Compare and contrast various forms of representations of patterns.
(A1D10) Understand and compare the properties of linear, quadratic and exponential functions (include domain and range)
(A1E10) Describe the effects of parameter changes on quadratic and exponential functions.
(G4B10) Draw or use visual models to represent and solve problems.

Essential Questions to Guide the Unit and Focus Teaching and Learning:

1. How do patterns help us represent, analyze, make predictions, and draw and justify conclusions from sets of data?
2. How can we use patterns to communicate mathematical ideas?

Specific Classroom Arrangement/Preparations:

Assigning students to groups is recommended.

Lesson Materials:

Folding paper

Copies of activity sheets for Lesson 4 from Appendix

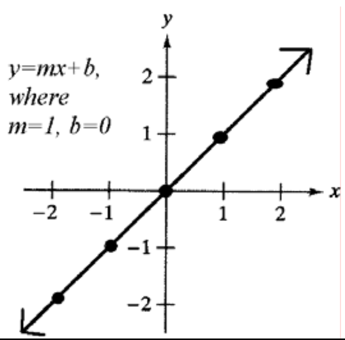
Lesson Four Assessment and Assessment Solutions

Technology/Manipulatives/Resources:

- Graphing Calculator (TI-83)

STEP-BY-STEP PROCESS:

Learning Activities	Student questions	Teacher Support
Warm Up: Each student should have a blank sheet of copying paper and a Data Recording Sheet for the Paper Folding Activity. They should complete the table on the Data Recording Sheet individually. Afterwards, they should work in small groups to complete the questions. A paper folding activity solutions page is provided.	**As you fold the paper in half each time what do you notice happening to the number of regions that are created?** Each time we fold the paper, the number of regions doubles.	Students should complete the table individually, then work in small groups to answer the questions as a whole class and discuss their findings. Students should notice the number of regions doubles with each new fold. The homework assignment for this lesson utilizes the table developed in this activity.
Patterns may be defined and extended by determining recursively the "Next" term. A recursive definition for a pattern consists of two parts. 1. An initial condition identifying where the sequence starts. 2. A recursive equation that explains how a term in a sequence can be found from the preceding term. Note: Most high school texts only allow the previous term, (a_{n-1}), to be the variable in the recursive equation, as we will	**In your own words, explain the difference between an explicit definition and a recursive definition for a pattern?** An explicit definition involves a formula for finding any term value when given the term number. A recursive definition tells you how to	Lessons 1-3 lead up to this lesson and cover linear, quadratic, and exponential functions. **The teacher should review/proceed to suit the needs of individual**

do here. Some college calculus texts allow both the previous term and the term number, (n), to be the variable in the recursive equation. An alternative way a pattern can be extended is to define the entire pattern explicitly. In this case you would determine the relationship between the term value and the term position number (i.e. the first term means n = 1, the second term means n = 2, etc.) and use the established *n* to determine subsequent terms in the pattern. When finding formulas for patterns we can write a recursive formula or an explicit formula. A recursive formula would have the initial condition, a_1, and a function, $a_n = f\left(a_{n-1}\right)$, for finding subsequent terms. An explicit formula is a function for the n^{th} term using only the term number *n* as the variable.	get successive terms in the pattern by being given a formula and the first term. The formula in the recursive definition leads you to the next term in the pattern.	**classes.** If students are not comfortable with the material in the first three lessons, especially if the first three lessons are not covered, then the initial definitions and examples are necessary parts of the instruction.
Linear Function – A linear function is a function that can be written in the form of $f(x) = mx + b$ where *m* and *b* are constants. The value *m* is called the slope and *b* is called the *y*-intercept. The graph of a linear function is a line. 	**What is a linear function?** Possible answers include the following. An equation in the form of $y = mx + b$. An equation whose graph is a line. A function whose rate of change is a constant.	**TEACHER NOTE:** Be sure to correct the misinformation if students refer to the variables as “not having exponents” in a linear function
Example 1: Extend the following patterns to find the next three terms. For each part determine both an explicit and recursive formula. Finally, graph the explicit formula and state the domain and range of the graph. **a. 5, 7, 9, 11, 13, ____, ____, ____**	**Describe in words the pattern you see in part a and use this pattern to determine the next three terms.** Possible answers include the following.	Arithmetic sequences can be tied into a discussion of linear functions.

Solution: 5, 7, 9, 11, 13, <u>15</u>, <u>17</u>, <u>19</u>
Recursive: Notice that we add 2 to move to the next term. This can be written as $a_1 = 5,\ a_n = a_{n-1} + 2$.

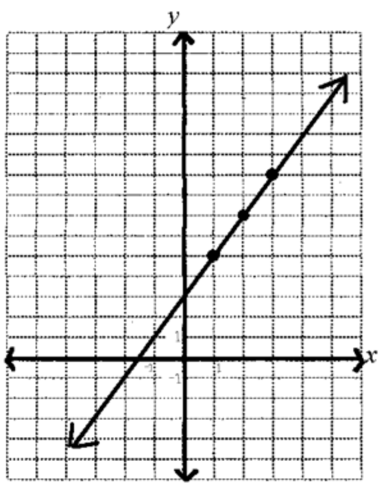
Explicit: Since we are adding the same value each time, we know that the explicit formula must be linear.

n or *x*	1	2	3	4
$f(x)$ or *y*	5	7	9	11

Using the formula $f(x) = mx + b$ and the points $(1,5)$ and $(2,7)$ (since the first term value is 5 and the second term value is 7) we can find that the slope is $m = \frac{7-5}{2-1} = \frac{2}{1} = 2$.

Plugging this in we have $f(x) = 2x + b$. Using the point $(1,5)$, we have $5 = 2(1) + b$, so $b = 3$. This gives the explicit formula $f(x) = 2x + 3$.

NOTE: The sequence represents a set of discrete data, and the line is a continuous function.



$y = 2x + 3$

Domain: all real numbers or $(-\infty, \infty)$
Range: all real numbers or $(-\infty, \infty)$

Each term is increasing by 2.
Odd numbers greater than 3.
The next terms are 15, 17, 19.

How can we identify the elements in the pattern with a "numerical position label"?
We have 5 as the first term (n = 1), 7 as the second term (n = 2), and so on.

In what ways can we now display this pattern?
i. e. We could put the term numbers and values in a table or we can graph them using the term number as the *x*-coordinate and the term value as the *y*-coordinate.

What information do we need in order to find the equation of a line?
In order to find the equation of a line we need the slope and a point.

What is the formula for the slope of a line?
$m = \frac{y_2 - y_1}{x_2 - x_1}$

Looking at the graph, what can we say about the determined pattern?
The pattern is increasing at a rate of change of 2 (slope).

Allow wait time before taking responses.

For example 1, work part a together on the board. Next, have students work part b on their own. After all have had a chance to work the problem, have them get into groups of 2-4 students to compare their responses. After students have finished comparing, have students put the answer and work on the board. You may decide to choose different groups to show their work for each of the parts of the problem.

You may want to have students interpret the graph. Specifically, they should look at the continuous graph vs the discrete data as well as the "left-hand" side of the graph ($x < 1$).
Steps to graphing $y = 2x + 3$
1. Press $y =$

<table>
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How do we find the domain and range of the linear function ?

The domain is the set of *x*-values that are defined for the equation and the range is the set of resulting *y*-values.

What given information will allow for the solution of a linear equation?
Information that will yield the slope and a point on the line.
</td>
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2. To the right of $y_1 =$, input $2x+3$
3. Press Zoom and then 6 (or Zoom Standard)
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b. 2, 5, 8, 11, 14, ____, ____, ____
Solution: 2, 5, 8, 11, 14, <u>17</u>, <u>20</u>, <u>23</u>
Recursive: Notice that we add 3 to move to the next term. This can be written as $a_1 = 2,\ a_n = a_{n-1} + 3$.
Explicit:

<table>
<tr><td>n or x</td><td>1</td><td>2</td><td>3</td><td>4</td></tr>
<tr><td>$f(x)$ or y</td><td>2</td><td>5</td><td>8</td><td>11</td></tr>
</table>

Since we are adding the same value each time, we know the explicit formula must be linear. Using the formula $f(x) = mx + b$ and the points $(1,2)$ and $(2,5)$ (since the first term value is 5 and the second term value is 7) we can find that the slope is $m = \frac{5-2}{2-1} = \frac{3}{1} = 3$. Plugging this in we have $f(x) = 3x + b$. Using the point $(1,2)$, we have $2 = 3(1) + b$, so $b = -1$. This gives the explicit formula $f(x) = 3x - 1$.

NOTE: The sequence represents a set of discrete data, and the line is a continuous function.
</td>
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Describe in words the pattern you see in part b and use this pattern to determine the next three terms.
One possible answer is that each term is increasing by 3.
The next terms are 17, 20, 23.

How can we identify the numbers in the pattern with a “numerical label”?
We have 2 as the first term (n=1), 5 as the second term (n=2), and so on.

In what ways can we now display this pattern?
We could put the term numbers and values in a table or we can graph them using the term number as the *x*-coordinate and the term value as the *y*-coordinate.
</td>
<td>

See above.

Steps to graphing $y = 3x - 1$

1. Press $y =$
2. To the right of $y_1 =$, input $3x - 1$
3. Press Zoom and then 6 (or Zoom Standard)
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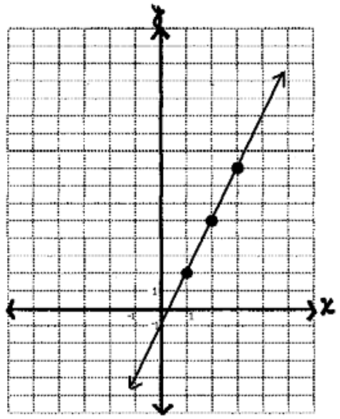
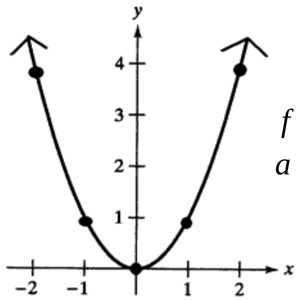
 $y = 3x - 1$ Domain: all real numbers or $(-\infty, \infty)$ Range: all real numbers or $(-\infty, \infty)$	**What pieces of information do we need in order to find the equation of a line?** In order to find the equation of a line we need information that will lead to the slope of the line and a point on the line. **What is the formula for the slope of a line?** $m = \frac{y_2 - y_1}{x_2 - x_1}$ **Looking at the graph, what can we say about this pattern?** The pattern is increasing and the rate of change is 3 (slope).	You may not need to ask for the formula again depending upon the memory level of your students. By asking for it again it will help reinforce the formula for the students.
Quadratic Function – A quadratic function is a function that can be written in the form of $f(x) = ax^2 + bx + c$ where *a*, *b*, and *c* are constants and $a \neq 0$. The graph of a quadratic function is a parabola.	**What is the shape of a quadratic function's graph?** One possible response is "body graphing". Students would simulate the parabola with raised, curved arms. **What must be included in order for an equation to be quadratic?** A quadratic is a polynomial of degree 2. **What is the equation of the pictured parabola?** $y = x^2$	"Body graphing" encourages the students to use their arms to demonstrate the shape of a graph. To show a parabola that opens up they would raise both arms above their head with their hands farther apart than shoulder width simulating the curve of a parabola.



$f(x) = ax^2 + bx + c$ where $a = 1$, $b = 0$, and $c = 0$

It is advisable to consider other quadratic functions where $b \neq 0$ and/or $c \neq 0$.

Example 2: Extend the following patterns to find the next three terms. For each part determine both an explicit and recursive formula. Finally, graph the explicit formula and state the domain and range of the graph.

5, 6, 8, 11, 15, ____, ____, ____

Solution: 5, 6, 8, 11, 15, 20, 26, 33

Recursive: Notice that we add 1 to the first term to get the second term. We then add 2 to the second term to get the third term. This pattern continues so that we add $n-1$ to the $(n-1)^{st}$ term to get the n^{th} term. This gives us the formula $a_n = a_{n-1} + (n-1)$.

Explicit:

n or *x*	1	2	3	4	5	6
$f(x)$ or *y*	5	6	8	11	15	20

Finite Differences:

1 2 3 4 5

1 1 1 1

Describe in words the pattern you see and use this pattern to determine the next three terms.

Possible answers include the following.

The terms are increasing by the pattern 1, 2, 3, and so on.

The next terms are 20, 26, 33.

How can we identify the numbers in the pattern with a "numerical label"?

We have 5 as the first term (n=1), 6 as the second term (n=2), and so on.

In what ways can we now display this pattern?

We could put the term numbers and values in a table or we can graph them using the term number as the *x*-coordinate and the term value as the *y*-coordinate.

When determining a formula that defines a sequence, we compute the finite differences to identify the degree of the equation. In a sequence that fits a linear formula, the first finite difference is a constant. In a sequence that fits a quadratic formula, the second finite difference is a constant.

Because of the 2^{nd} row of finite differences this function must be a quadratic.		In the given example the first row of finite differences gives us 1, 2, 3, 4 and so on. The second row of finite difference is the constant 1. Therefore, the function in question is quadratic.
Extension: Using the formula $f(x)=ax^2+bx+c$ and plugging in $(1,5)$, $(2,6)$, and $(3,8)$ we can find an explicit formula. $\begin{cases} 5=a+b+c \\ 6=4a+2b+c \\ 8=9a+3b+c \end{cases} \rightarrow \begin{cases} 1=3a+b \\ 2=5a+b \end{cases} \rightarrow 1=2a$ $\rightarrow a=\frac{1}{2},\ b=\frac{-1}{2},\ c=5$ The explicit formula is $f(x)=\frac{1}{2}x^2-\frac{1}{2}x+5$. ***NOTE: The sequence represents a set of discrete data, and the quadratic is a continuous function.***	**What information do we need in order to find the equation of a quadratic function?** One method to find the equation of a quadratic function is to identify three points from the data set. Create a system of 3 equations with 3 unknowns (a, b, and c), and solve the system. **What three points could we use?** $(1,5)$, $(2,6)$, and $(3,8)$	By using these three points, we have created a system involving three equations and three unknowns. It can be solved a variety of ways • Substitution elimination • Augmented matrices • Inverse matrices on the calculator To use a graphing calculator (TI-83/84) you will enter the coefficients and the constant term into matrices. The method of doing this is listed below: 1. Hit 2^{nd} then x^{-1}

		(matrix) 2. Arrow over to EDIT and then go to 1:A and press enter (this names our matrix A) 3. Input the dimensions of the coefficient matrix, 3×3, causing 3 rows and 3 columns of zeros to appear 4. Arrow down and input the coefficients from the 3 equations in place of the zeros and hit enter after each number $\begin{bmatrix}1 & 1 & 1\\4 & 2 & 1\\9 & 3 & 1\end{bmatrix}$ 5. Hit 2nd then x^{-1} (matrix) 6. Arrow over to EDIT and then go to 2:B and press enter (this names our matrix B) 7. Input the dimensions of the coefficient matrix, 3×1, causing 3 rows and 1 column of zeros to appear 8. Arrow down and input the 3 numbers from the left hand side of the 3 equations in

		place of the zeros and hit enter after each number $\begin{bmatrix} 5 \\ 6 \\ 8 \end{bmatrix}$ 9. Now hit 2nd MODE to go back to the main screen (quit) 10. Hit 2nd then x^{-1} (matrix) 11. Under Names, press enter on 1:A to bring it up on the main screen 12. Now hit x^{-1} so that it can take the inverse of the matrix A (don't hit enter yet) 13. Hit $\times$ key (multiplication) 14. Hit 2nd then x^{-1} (matrix) 15. Under Names, press enter on 2:B to bring it up on the main screen 16. Now hit enter 17. The result should be a 3×1 matrix with the **values of *a*, *b*, and *c***

$\begin{bmatrix} .5 \\ -.5 \\ 5 \end{bmatrix}$ to give

We get the equation:

$y=\frac{1}{2}x^2-\frac{1}{2}x+5$

The vertex of the parabola is at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ and

$\frac{-b}{2a}=\frac{-(-1/2)}{2(1/2)}=\frac{1}{2}$. So $f\left(\frac{1}{2}\right)=\frac{1}{2}\left(\frac{1}{2}\right)^2-\frac{1}{2}\left(\frac{1}{2}\right)+5=\frac{39}{8}=4\frac{7}{8}$.

Thus, the vertex of the parabola is at $\left(\frac{1}{2},4\frac{7}{8}\right)$. We also can obtain points that are on the opposite side of the parabola by using the points in the table above and reflecting them through the line $x=\frac{1}{2}$ (the axis of symmetry for this parabola).

x	-2	-1	0	$\frac{1}{2}$	1	2	3
$f(x)$ or y	8	6	5	$4\frac{7}{8}$	5	6	8

What do you need in order to graph a parabola?
Students may say several points, but encourage them to specifically state they need the vertex and at least one point on each side of the vertex.

How can we find the vertex of a parabola from its equation?
$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$ or $\left(\frac{-b}{2a}, y\right)$, or we can complete the square in the quadratic.

Alternatively, we could complete the square to put the equation in graphing form $y=a(x-h)^2+k$ where (h,k) is the vertex.

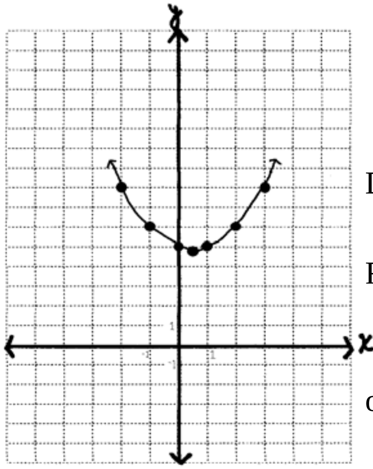
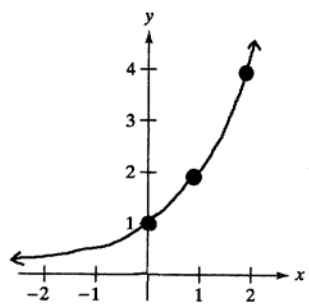
What is the vertex of the parabola?
$\left(\frac{1}{2},4\frac{7}{8}\right)$

Have a student go to the board or overhead to calculate the vertex.

Looking at our chart with the *x* and *y* values, notice that we do not have any points to the left of our vertex. How can we get these points?
The students should recall that a parabola is symmetric with respect to the axis of symmetry. Using this they

Steps to graphing $y=\frac{1}{2}x^2-\frac{1}{2}x+5$

1. Press $y=$
2. To the right of $y_1=$,

 $y=\frac{1}{2}x^2-\frac{1}{2}x+5$ Domain: all real numbers or $(-\infty,\infty)$ Range: all real numbers y such that $y\geq 4\frac{7}{8}$ or $\left[4\frac{7}{8},\infty\right)$ Note: the use of interval notation!	can get additional points to the left of the vertex. **How do we find the domain and range?** One possible response would be that the domain is the set of *x*-values that are defined for the given function (all real numbers) and the range is the set of resulting *y*-values.	input $\frac{1}{2}x^2-\frac{1}{2}x+5$ 3. Press Zoom and then 6 (or Zoom Standard)
An exponential function is written in the form of $f(x)=ab^x$ where *a* and *b* are constants, $a\neq 0$, and $b>0$.  $f(x)=ab^x$ where $a=1$ and $b=2$	**What does an exponential function look like when we graph it?** One possible response is "body graphing" where the students would raise one arm (slightly curved) and hold the other arm nearly horizontal to form the shape. **What is apparent in the equation of an exponential function?** The variable should be in the exponent. **What is the equation of the pictured exponential function?** $y=2^x$	See "body graphing" information above. Discussing additional exponential functions with different a and b values is recommended. Consider working with an exponential where 0<b<1.
Example 3: Extend the following patterns to find the next three terms. For each part determine both an explicit and recursive formula. Finally, graph the explicit formula and state the domain and range of the graph.	**Describe the pattern in the given sequence and determine the next three terms.** Possible answers include the	This is a good time to remind students of the geometric sequence.

1, 3, 9, 27, ____, ____, ____

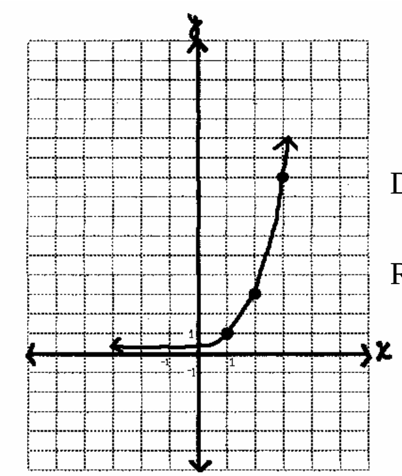
Solution: 1, 3, 9, 27, <u>81</u>, <u>243</u>, <u>729</u>

Recursive: Notice that we multiply by 3 to move to the next term. This can be written as $a_1 = 1$, $a_n = 3a_{n-1}$.

Explicit:

n or *x*	1	2	3	4
$f(x)$ or *y*	1	3	9	27

Noting that the terms have a common ratio of 3, our formula should have 3^x as a part of it. Term number 1 has a value of 1, so we have the explicit formula $f(x) = 3^{x-1}$.



$y = 3^{x-1}$

Domain: all real numbers or $(-\infty, \infty)$

Range: all positive real numbers $y > 0$ or $(0, \infty)$

NOTE: The sequence represents a set of discrete data, and the exponential is a continuous function.

following.
Each term is being multiplied by 3.
The terms are powers of 3.
The next terms are 81, 243, 729.

How can we identify the numbers in the pattern with a "numerical position label"?
We have 1 as the first term (n=1), 3 as the second term (n=2), and so on.

In what ways can we now display this pattern?
We could put the term numbers and values in a table or we can graph them using the term number as the *x*-coordinate and the term value as the *y*-coordinate.

What information do we need in order to find the equation of an exponential function?
In order to find the equation of an exponential function, we need to know the base and the leading coefficient. The base can be found by identifying the constant ratio of a_n/a_{n-1}. This example has 3 as that ratio. Using the base and an exponent of *x*, the leading coefficient can be determined.

Looking at the graph, what can we say about this pattern?

Steps to graphing $y = 3^{x-1}$ on the calculator:

1. Press $y =$
2. To the right of $y_1 =$, input $3 \wedge (x-1)$
3. Press Zoom and then 6 (or Zoom Standard)

To further study exponential functions, use the activities at the end of this lesson. The worksheets are in the **Appendix.**

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<td>This is an increasing function, and the rate of increase seems to be increasing.

How do we find the domain and range?
One possible response would be that the domain is the set of x-values that are defined by the exponential (all real numbers) and the range is the set of resulting y-values.</td>
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<td>Example 4: Bobbie has set up a stand to sell pies for $5 each. For each situation, complete the following table to find the total number of pies sold and the amount of money he collects. Assuming there was a total of 10 customers on the given day. For each day you will also need to determine a recursive formula and an explicit formula for the number of pies sold and the price of the pies for the nth customer.
a. On the first day of selling pies, each customer buys the same number of pies as his customer number.
<table>
<tr><th>Customer Number</th><th>Number of Pies Sold</th><th>Cost of Pie(s)</th></tr>
<tr><td>1</td><td></td><td></td></tr>
<tr><td>2</td><td></td><td></td></tr>
<tr><td>3</td><td></td><td></td></tr>
<tr><td>4</td><td></td><td></td></tr>
<tr><td>5</td><td></td><td></td></tr>
<tr><td>6</td><td></td><td></td></tr>
<tr><td>7</td><td></td><td></td></tr>
<tr><td>8</td><td></td><td></td></tr>
<tr><td>9</td><td></td><td></td></tr>
</table>
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<td>For Example 4 (a, b, and c) refer to the Appendix to provide tables to the students..

Assign students to cooperative groups for this problem (see Lesson 1 for structure of Cooperative Group work). At the end of solving the problem, different groups should be called upon to present their findings.

The teacher should the work being done by the</td>
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</table>

10		
total		

Solution:

Customer Number	Number of Pies Sold	Cost of Pie(s)
1	1	$5
2	2	$10
3	3	$15
4	4	$20
5	5	$25
6	6	$30
7	7	$35
8	8	$40
9	9	$45
10	10	$50
total	55	$275

Since the number of pies sold to each customer is the same as the customer number, we have the explicit formula that $a_n = n$, where a_n is the number of pies and *n* is the customer number. We can also notice that the number of pies increases by one each time so $a_n = a_{n-1} + 1$, where a_n is the number of pies, is the recursive formula for the number of pies sold.

To obtain the explicit formula for cost of the pies sold, we can multiply the number of pies sold by 5. This gives us $a_n = 5n$, where a_n is the cost of the pies sold and *n* is the customer number. Looking at a recursive pattern, we notice that the price increases by $5 each time, which means that we are adding 5 each time. This gives a recursive formula of $a_n = a_{n-1} + 5$ for the price of pies, where a_n is the price of the

What type of function did you find for relating the customer number and the number of pies sold? What type of function did you find for relating the customer number to the price of the pie? Why did you choose that function?

The functions were both linear. Some mention might be made to slope or change in “y” compared to the change in “x” being constant in each relationship.

What differences did you notice between the formula for the number of pies sold and the formula for the cost of the pies sold?

One is y = x and the other is y = 5x.

The cost of the pies sold formula was the number of pies sold multiplied by 5.

groups and answer questions as needed.

pies. Summing the columns, we can see that he sold 55 pies and earned $275.		
b. On the second day of selling pies, the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 7 pies, the fifth customer buys 11 pies, and so on.	**What type of function did you find for relating the customer number and the number of pies sold?** **What type of function did you find**	

Customer Number	Number of Pies Sold	Cost of Pie(s)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
total		

Solution:

Customer Number	Number of Pies Sold	Price of Pie(s)
1	1	$5
2	2	$10
3	4	$20
4	7	$35
5	11	$55
6	16	$80
7	22	$110
8	29	$145
9	37	$185
10	46	$230
total	175	$875

Since the number of pies sold to each customer follows the pattern 1, 2, 4, 7, and continues to increase by one additional pie for each customer, we know the explicit formula is quadratic (constant difference in second row of finite differences). Using the points $(1,1)$, $(2,2)$, and $(3,4)$ as well as the formula $f(x)=ax^2+bx+c$, we can find the explicit formula is $a_n=\frac{1}{2}n^2-\frac{1}{2}n+1$, where a_n is the number of pies and *n* is the customer number.

$$\begin{cases}1=a+b+c\\2=4a+2b+c\\4=9a+3b+c\end{cases}\rightarrow\begin{cases}1=3a+b\\2=5a+b\end{cases}\rightarrow 1=2a$$

$$\rightarrow a=\frac{1}{2},\ b=\frac{-1}{2},\ c=1$$

For the recursive formula we notice that we are adding the previous term value and the previous term number to find the next term. In other words, $a_n=a_{n-1}+(n-1)$, where a_n is the

for relating the customer number to the cost of the pies sold? Why did you choose that function?

Both functions are quadratic.
By finite differences we can see the functions are both quadratic

What differences did you notice between the formula for the number of pies sold and the formula for the cost of the pies sold?
The price of pies formula was the number of pies sold multiplied by 5.

How do you solve for a, b, and c to determine the quadratic function?

Students should recall the solution methods are
1) elimination/substitution,
2) augmented matrices, and
3) calculator matrices.

Teacher Note: Students should be encouraged to become adept in using all 3 methods—systems, matrices, and calculator matrices—of finding a, b, and c for the quadratic.

number of pies and *n* is the customer number.

To obtain the explicit formula for cost of the pies sold, we can multiply the number of pies sold by 5. This gives us $a_n = \frac{5}{2}n^2 - \frac{5}{2}n + 5$, where a_n is the cost of the pies sold and *n* is the customer number. Looking at a recursive pattern, we notice that the cost if the pies sold follows the pattern 5, 10, 20, 35, and continues to increase by $5 times the previous term number for each customer. This gives a recursive formula of $a_n = a_{n-1} + 5(n-1)$ for the price of pies, where a_n is the price of the pies and *n* is the customer number.

Summing the columns, we can see that he sold 175 pies and earned $875.

c. On the third day of selling pies the first customer buys 1 pie, the second customer buys 2 pies, the third customer buys 4 pies, the fourth customer buys 8 pies, and so on.

Customer Number	Number of Pies Sold	Cost of Pie(s)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
total		

Solution:

What type of function did you find for relating the customer number and the number of pies sold? What type of function did you find for relating the customer number to the cost of the pies cold? Why did you choose that function?

The functions were both exponential.

Customer Number	Number of Pies Sold	Cost of Pie(s)
1	1	$5
2	2	$10
3	4	$20
4	8	$40
5	16	$80
6	32	$160
7	64	$320
8	128	$640
9	256	$1280
10	512	$2560
total	1023	$5115

Since the number of pies sold to each customer doubles each time, we have the explicit formula that $a_n = 2^{n-1}$, where a_n is the number of pies and *n* is the customer number. We also have that the recursive formula is $a_n = 2a_{n-1}$, where a_n is the number of pies.

To obtain the explicit formula for price of the pies, we can multiply the number of pies sold by 5. This gives us $a_n = 5\left(2^{n-1}\right)$, where a_n is the price of the pies and *n* is the customer number. Looking at a recursive pattern, we notice that the price column still doubles each time. This gives a recursive formula of $a_n = 2a_{n-1}$ for the price of pies, where a_n is the price of the pies.

Summing the columns, we can see that he sold 1023 pies and earned $5115.

For each step of 1 in the number of customers there was a common constant ratio multiplied by the "y" value to get the next "y" value.

What differences did you notice between the formula for the number of pies sold and the formula for the cost of the pies sold?
The cost of the pies sold formula was the number of pies sold multiplied by 5.

What comparisons can you make between the solutions for parts a, b, and c?

When the change in x is 1:

Part a (linear) the change in y is a constant (slope).

Part b (quadratic) change in y increases by differing amounts (not constant). However, the second finite difference is constant.

Part c (exponential) The y-values are multiplied by a constant ratio to get the succeeding y-value.

As an **extension,** students can graph the function relating the number of pies sold and the price of the pies for each of the parts a, b, and c and then compare the three graphs.

Activity 1 – Interior Angle Sum Activity

See Interior Angle Sum Activity sheet for questions to ask the students.

This activity (contained in the **Appendix**)

During this activity students should be working in small groups to complete the activity page. They will be extending a pattern, finding a recursive and an explicit formula for the pattern, and graphing the data. The activity also has the students determine what restrictions must be placed on the formulas so that the resulting equation models the physical data. **After completion of the activity the teacher should lead the class in discussing their findings.**		generates data that is linear. A graphing calculator can be used to find the linear regression formula for the data. See handout for sample solutions page (contained in the **Appendix**). Directions for using the calculator are included within the solutions. The teacher should monitor the groups and answer questions.
Activity 2 – Number of Diagonals in a Polygon Activity During this activity students should be working in small groups to complete the activity page. They will be extending a pattern, finding a recursive and an explicit formula for the pattern, and graphing the data. The activity also has the students determine what restrictions must be placed on the formulas so that the resulting equation models the physical data. **After completion of the activity the teacher should lead the class in discussing their findings.**	See Number of Diagonals in a Polygon Activity sheet (see **Appendix**) for questions to ask the students.	This activity (contained in the **Appendix**) generates data that is quadratic. A graphing calculator can be used to find the quadratic regression formula for the data. See handout for sample solutions page (contained in the **Appendix**). Directions for using the calculator are included within the solutions. The teacher should monitor the groups and answer questions
Activity 3 – M&M's Activity	See the M & M's Activity sheet (in the	This activity (contained in

Complete the M & M's Activity Page. During this activity students should be working in small groups to complete the activity page. They will be doing an experiment and collecting data. They will then use the calculator to input the data, graph the data, and find the equation of best fit for the data. **After completion of the activity the teacher should lead the class in discussing their findings.**	**Appendix**) for questions to ask the students.	the **Appendix**) generates data that is exponential. A graphing calculator can be used to find the exponential regression formula for the data. See handout for sample solutions page (contained in the **Appendix**). Directions for using the calculator are included within the solutions. The teacher should monitor the groups and answer questions